Quantum Computing and Schrödinger's Cat

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Abstract

I have been greatly honoured in receiving the ETH Zürich Latsis prize in 2016. The following constitutes a review of a part of my work performed in the years leading to the prize, and I hope it gives a feeling of the long-term motivations, and the beauty of being able to perform experimental research investigating the basic notions of quantum science.

1 Introduction

The counter-intuitive rules of quantum mechanics allow computations which lie outside the capabilities of any supercomputer which relies only on classical physics. Tasks with a quantum advantage include calculations of electronic structure in molecules and materials as well as codebreaking. Building a quantum computer requires precisely controlling individual quantum systems and using them to store and manipulate information. As our control increases, it provides possibilities to probe quantum mechanics in regimes which were previously inaccessible. One example is the boundary between classical and quantum physics, where demonstration of large quantum systems can be considered to be analogous to paradoxical situations such as Schrödingers dead and alive cat. The experiments described in the following constitute work performed in an apparatus designed for implementing small-scale quantum computing logic circuits. Working with a single trapped atom in this system, we have pursued the creation and manipulation of quantum states which are analogous to those found in the Schrödinger's cat thought experiment. This control is directly applicable to our work on quantum computation.

2 Trapped ions and quantum physics

My work explores quantum computing using individual atoms with one electron removed - these are then charged atomic ions. Due to the charge, they can be trapped by combinations of static and oscillating electrical fields in ultra-high vacuum. Quantum bits (qubits) are stored by choosing two different states of the least strongly bound electron in the atom. The electron has a higher energy if it points up (the same direction as the magnetic field in our laboratory) than it does if it points down. As a short hand, we write these states as $|\uparrow\rangle$ and $|\downarrow\rangle$. If the electron is in the $|\uparrow\rangle$ state, we say that the logical state is "1", and if the electron is in the $|\downarrow\rangle$ state, we say that the logical state is "0". These electron states are very stable - an ion initialized into one of these stays in it for years. This means that a bit of information can be stored for a long time. By shining a laser on our atom, we can find out whether the electron is $|\uparrow\rangle$ or $|\downarrow\rangle$. In the first case, the ion scatters lots of light (some of which we can collect), while in the other, the light passes straight through the ion without interacting with the electron. Therefore, if we see light, we know the logic is “1”, but if we don’t, we know that the logic is “0”.

Quantum computing deals with an extra possibility, which is that the electron can exist in a *superposition* in which we have to think of it as pointing in both directions simultaneously. Here an additional factor called the phase becomes critically important. To see the phase relationship, we would measure not whether the electron points up or down, but whether it points left or right. If we prepare a series of electrons all in $|\uparrow\rangle$ and we measure each along the right/left axis, we will find that the result is not predictable, ie. we will find the answer “left” half of the times that we perform the experiment, and the answer “right” on all other occasions. But we would find exactly the same if we performed the same experiment with electrons prepared in the state $|\downarrow\rangle$. How can we differentiate between the two? For quantum mechanics, the two states are written as

$$|\uparrow\rangle = \frac{1}{\sqrt{2}}\left(|\leftarrow\rangle + |\rightarrow\rangle\right) \tag{1}$$
$$|\downarrow\rangle = \frac{1}{\sqrt{2}}\left(|\leftarrow\rangle - |\rightarrow\rangle\right)$$

thus the difference between the two is in the sign, which gives a relationship between the two possible states which we measure. Note that we determine this sign by measuring whether the particle points “up” or “down”. In quantum physics, whenever we have a system in a state where there is equal probability to get one of two measurement outcomes, we have to write the state as

$$\frac{1}{\sqrt{2}}\left(|\text{“state corresponding to outcome 1''}\rangle + e^{i\phi}\,|\text{“state corresponding to outcome 2''}\rangle\right). \tag{2}$$

The $e^{i\phi}$ is a complex number with a definite phase ϕ which is somewhere between 0 and 360 degrees - we can view it as a position on a clockface which is intrinsic to the atom. When $\phi = 0$, $e^{i\phi} = 1$, while when $\phi = \pi$, $e^{i\phi} = -1$ - these are the two special cases given for our atom above. The relationship to clocks is highly prescient - atomic clocks which define the time we use today are stabilized by referencing the second to the quantum phase of an atom which evolves in a very predictable manner as time advances. However phase is also a well known aspect of waves - when the phase relationship between two waves is well defined, then they *interfere*. In the case of the quantum, where the phase is a relationship between two possibilities then seeing interference requires that the two possibilities existed simultaneously. Only when quantum mechanical states have a well-defined phase are they useful for quantum computers, since the phase relationship allows interference effects which are key to efficient information processing.

The need for systems to be simultaneously in two different states in order to interfere is counter to our perception of the world which has accompanied us from childhood. It was considered a major interpretational problem by the founders of quantum mechanics. Erwin Schrödinger illustrated the apparently nonsensical situation through a thought-experiment which has become known as the “Schrödinger’s cat” paradox. This paradox is illustrated in figure 1 and results in the problematic conclusion that everyday objects such as cats can be put into superposition states, ie. they should be dead and alive simultaneously. I don’t think that any of us has observed such a cat, rather we either see one which is dead or alive. This poses the question, where does the transition between the classical and quantum regimes lie? When, and for what type of objects do we only see one option at any one time, and when can they be in two states at the same time? On the path to a quantum computer, the increased control we gain over quantum systems such as atoms in the laboratory allows us to probe this boundary.

When we shine laser light onto our atom with the correct frequency, the atomic electron is gradually transferred from one state to the other, such that the probability to find the electron in $\uparrow$ or $\downarrow$ varies sinusoidally with time. If we turn the laser off halfway between these two extremes, we make the superposition state

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle + |\downarrow\rangle \right) \ . \tag{3}$$

and if we measured whether the electron pointed right or left, we would always get the answer “right”. With a different choice of the phase of the laser, we can create the state

$$|\leftarrow\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\rangle - |\downarrow\rangle \right) \ . \tag{4}$$

which would result in the opposite result for the measurement; we would always find the answer “left”. These equations will be useful in our later discussion.

But in our lab it is not just the electrons of our atomic ions which show quantum behavior. The ion is cooled to around 1-millionth of a degree Kelvin by laser light, at which point the motion of the atom also shows discrete energies which are characteristic of bound quantum sytems. The motion is specified by the position and speed of the atom, and in what follows I will present the discussion in terms of position. The laser light affects the position of the ion because light carries a force, which pushes the atom.

By turning on a laser with two frequencies simultaneously, we create a situation in which the light pushes the atom to the left if it is in the state $|\leftarrow\rangle$, while it pushes the $|\rightarrow\rangle$ state right. We call this a state dependent force. If we do this starting from a situation where the spin is pointing $\uparrow$ (which is a superposition of $|\leftarrow\rangle$ and $|\rightarrow\rangle$, as written in equation 1), we create a state in which the electron logic is *entangled* with the position of the atom. The state would be written

$$|\psi_{\rm cat}\rangle = \frac{1}{\sqrt{2}} \left(|\leftarrow\rangle \, |\text{left}\rangle + e^{i\theta_q} \, |\rightarrow\rangle \, |\text{right}\rangle \right) \tag{5}$$

Each of the possibilities for the measurements now involves both the electron and the atom position, ie. $\leftarrow$ *and* “left” or $\rightarrow$ *and* “right”. The interpretation of such a state also says that the atom exists both on the left and right sides at the same time. If these positions are separated by a large distance they start to come close to the size scales which we would consider “classical”. At this point we start to worry that the quantum system is showing its behaviour in a regime we associate with classical physics, exactly the point that Schrödinger was illustrating with his cat paradox. As a result these states have become known as “Schrödinger’s cat” states. To test how far our control of the world allows us to extend quantum mechanics into a regime which we consider to be the “large-scale” domain of classical physics, we try to create large states, ie. increase the distance between left and right as much as possible. The states created recently in my laboratory are the largest such states created until now. They are separated by distances of up to 330 nm, which for the first time is large enough that the two positions could be resolved using an optical microscope.

How do we know that we create a “cat” state? The measurement we make is to find out whether the spin is $\uparrow$. We do this by shining another frequency of laser light onto the ion. If the ion is in the $|\uparrow\rangle$ state, the ion interacts with the light and scatters light into all directions. We pick up some of these on a detector. In the other case, when the ion is in $|\downarrow\rangle$, the ion does not interact with the light and no light arrives in our detector.

If we make such a measurement, we will always get $\uparrow$ as the result if there is a good phase relationship between $|\leftarrow\rangle$ and $|\rightarrow\rangle$. This is the case before the force has been applied, but once we have applied the force for some time, the state $|\leftarrow\rangle$ is in a different place in space to the state $|\rightarrow\rangle$. This means that the two possibilities can not interfere. A more technical way to say this is that the phase of the entangled state is a property of the correlation between spin and position, and not accessible through either on its own. Data from an experiment where the states are gradually separated as a function of time is shown in figure 2. For short times, we see that the probability to find $\uparrow$ is high, but this reduces as the position

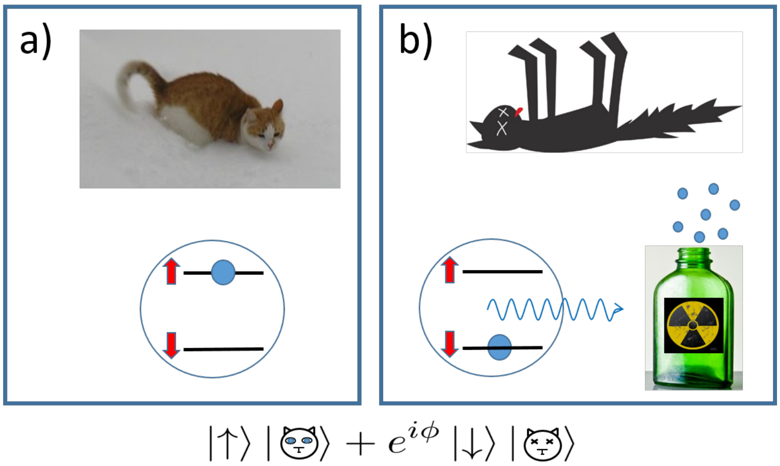

Figure 1: **The Schrödinger's cat paradox:** a) Butler the cat is alive, and is inside a box along with an atom for which the electron is pointing up. b) at some (unknown point in time), the electron switches from up to down. This requires the atom to emit light, which cracks open a vial of poison, all of which is swallowed by the cat and which poisons it. If this all happens inside a closed box, then before the box is opened the correct description of the state of the cat + atom is to write a superposition, complete with a quantum mechanical phase. But we never see interference effects for cats, we only encounter a dead cat, or one which is alive. No cats are harmed in our experiments. As of 2016, Butler was still alive and living with my old flatmate in Billingham, USA.

and electron spin become entangled. At long times, we have a probability of 1/2 to find the spin in $\uparrow$, which means that there is no interference between $\leftarrow$ and $\rightarrow$ any longer. Whether the positions can be considered to be separate depends on the size of the region that the ion occupies. In our experiments we can tune this, and in the figure you see three cases where we have prepared the ion in a different size of state prior to applying any force.

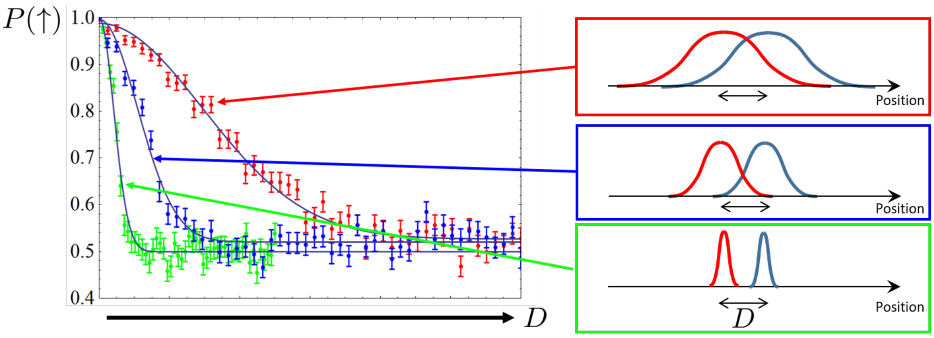

Figure 2: **Loss of interference due to spatial separation:** As the state-dependent force is applied for an increasing time, the position of the atom separates, depending on the spin state of the electron. The distance between these two is given as the D axis of the graph. For large separation this means that the $|\rightarrow\rangle$ and $|\leftarrow\rangle$ states can no longer interfere with each other. The probability to find the electron $\uparrow$ is 1 if these states fully interfere, but 1/2 if they cannot. The data show this transition, for three cases in which we prepared three different "widths" of the atom originally. The interference is lost at smaller separations when we make the starting state narrower, a situation which is called a "squeezed" initial state.

A further feature which we wanted to show in our experiments was that the separated states are related by a definite phase. To do this, we made use of a special feature of quantum measurement, which is that the act of obtaining information changes the state of the atom, even if no physical interaction happens. Let us take another look at the measurement, but now paying attention to what happened to the position of the atom if we find the atom in one or the other electronic state. To look at this we should write $|\psi_{\text{cat}}\rangle$ in terms of the states which describe the results of the measurement, which could either be $\uparrow$ or $\downarrow$. This is done by replacing $|\leftarrow\rangle$ and $|\rightarrow\rangle$ with the formulae for superpositions which you see in equations 4 and 3. We find

$$|\psi_{\text{cat}}\rangle = \frac{1}{2}\,|\uparrow\rangle\left(|\text{left}\rangle + e^{i\theta_q}\,|\text{right}\rangle\right) + \frac{1}{2}\,|\downarrow\rangle\left(|\text{left}\rangle - e^{i\theta_q}\,|\text{right}\rangle\right) \tag{6}$$

When we get a measurement result $\downarrow$, we select out the second part of this equation, although no light was scattered from the atom. The new wavefunction for the atom is then

$$|\uparrow\rangle\left(|\text{left}\rangle - e^{i\theta_q}\,|\text{right}\rangle\right) \tag{7}$$

Now the phase θ_q only belongs to the position of the atom, there is no dependence on the spin. This means that by making a suitable measurement of the position of the atom, we should be able to see the phase relationship, and show that two different positions really are related by this hallmark feature of quantum mechanics. The measurement we make is of the energy of the atom. At this point I have to state that in all that I described up until now, I made a gross simplification of what our atom does. The atom is trapped, and instead of being "left" or "right" it makes oscillations about the centre of the trap. When I write "left" I am imagining that I only look at certain points of time, when the atom is on the left side. This happens once every cycle of oscillation. At that point in time, the "right" state would have the atom on the right hand side. Why is the fact that it oscillates now important? In quantum mechanics, the energy of an oscillator can only take certain discrete values, which are multiples of hf, where h is a constant and f is the frequency of oscillation. When we make many repeats of an experiment, and measure the energy each time, we get each of these discrete values with a certain probability. For the cat state $|\psi_{\text{cat}}\rangle$ you can see our measured values in figure 3. If we do the same energy measurement after the prior measurement of whether the spin is $\uparrow$ or $\downarrow$, we see that the energies have changed dramatically. In the first case, all the energy values with an odd multiple have zero probability of being occupied. In the second case, it is the energy values with an even multiple which have zero probability of being occupied. This "deletion" of accessible energies is due to interference of the two separated positions of the atom - the two possibilities interfere while being at two separate locations.

The result that light which doesn't interact with the atom can nevertheless completely change the quantum mechanical state is an illustration of one of the most fundamental points of quantum measurement. The measurement in this case involves no light being scattered from the atom, ie. the laser which we use for detection passes straight through the atom without being modified. Nevertheless the quantum state of our atom completely changed, such that where before it could take any of the possible values of energy, after the measurement it can only take values with even multiples. This sudden change in the state due to a change in the state of knowledge of the observer is at the heart of many of the strange features of quantum mechanics, and it is a key feature of the new possibilities for using quantum systems in information processing. One view of a quantum information processor is that the interference which results from a carefully chosen measurement is such that only the correct answer to the problem that you are trying to solve has any probability of being measured. All of the other possible outcomes interfere in such a way that they have no possibility to occur.

The experiments above work on trying to see quantum effects at larger length scales, but they are also demonstrating the principal ingredients required for ion trap quantum computing. Current designs for a quantum computer based on trapped ions make use of the electron states to store the fundamental units of information. However the motion of the atom

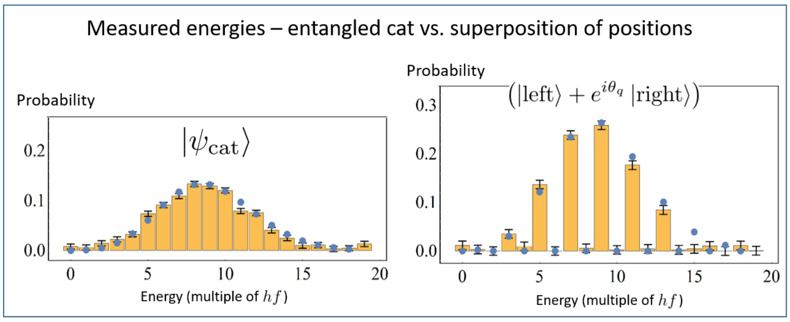

Figure 3: **Observation of interference between remote locations:** Left) The energies of the oscillation of the atom for the state $|\psi_{\rm cat}\rangle$. The height of each bar is the observed probability to find the atom at that particular energy. Right) The energies of the oscillation of the atom if we first measure the electron to be $\uparrow$, and then measure the energy. After the electron measurment, value of the phase becomes well defined as $\theta_q = \pi$, and the state is $|\text{left}\rangle - |\text{right}\rangle$. This results in interference between the two separate spatial possibilities such that the probability to find the atom at an even multiple of the basic unit of energy hf is zero. This effect is one of the hallmarks of quantum physics, and is strange because the light used for the measurement of the spin of the electron passed through the atom without interacting with it. Nevertheless, the quantum state of the atom changed dramatically, deleting all the even energy possibilities.

itself is critical to performing logical operations between different qubits. We use the state-dependent forces to make the ion move to the right or left dependent on the spin. Another ion would then feel a repulsion which increases or decreases dependent on whether the first ion came closer or moved away. This communication via the electrostatic repulsion of the two charged atoms allows conditional operations to be performed between two bits stored in each atom: these interactions are the crucial step to building a quantum computer. In our laboratory, we have realized these interactions at precisions which are now high enough that a reliable quantum computer could cope with the residual errors and thus could be built. The test bed for the precision of our control was the experiments controlling Schrödinger's cat states that your read about in the text above.

3 Future directions

A quantum computer is likely to involve more than 1000 quantum bits, but this would actually require over 100,000 ions. The reason for the large difference between ion number and the quantum bit number is that we don't perform every manipulation perfectly. In order to cope with this, we will have to make use of quantum methods of error-correction codes. These rely on entangled states to encode each quantum bit. The smallest example code for a single qubit uses 7 ions. Manipulating first one and then two of these encoded entangled qubits is the current status of work in quantum computing. This work provides insights into the systems we use, and the relationship between entangled states and noise, the latter of which is unavoidable at some level in real physical systems. This work is ongoing, but we see no fundamental problem hindering the development of a quantum computer. Despite this positive statement, the technical challenges look both daunting and inviting, and I wouldn't currently want to predict when a useful quantum computer would become operational.

Scaling up will hit a very interested level at system sizes of around 40 ions. At this level, the quantum state of the system cannot be encoded onto any supercomputer of the type we have today, because it would require 2^{40} numbers to be stored simultaneously. Computing how the system may change in time is even more challenging. The reason we can sometimes simulate systems of more than 40 quantum particles in practice is because there exist special cases in which the particles only interact in a limited way, or where there is a lot of symmetry. Since we can engineer interactions with our ions, we can steer them outside these simple cases. This will take us to a complexity boundary of modern physics, which neither experiment or simulation has ever been before.