Location planning in distributive Systems – Models, Methods, Applications

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Introduction

Decisions about the distribution system are a strategic issue for almost every company. The problem of locating facilities (sourceing points, production plants, distribution centers, depots, and transshipment points) and allocating customers covers the core components of distribution system design. Within supply chain management, locational planning does not only involve the geographic placement of facilities but the design of the complete distribution system or supply chain network. Interdependent decisions which have to be taken in distribution system design are to determine

- the number of distribution stages,
- the size, number and location of facilities,
- the allocation of customer demands to depots,
- the paths and quantities of product flows from sourceing points to the final customer

in such a way that customer requirements can be met efficiently. These decisions are of great significance to a firm since they represent the basic strategy for accessing customer markets, and will have a considerable impact on revenue, cost, and level of service.

Location-Allocation Models

Due to their long-term and strategic nature and the considerable impact on a firm's competitiveness, locational decisions require careful planning based on quantitative modelling tools which allow to optimise the distribution system structure and to assess the impact of changes in cost rates, capacities and customer demands on total distribution cost and service levels.

Mathematical models which address location-allocation problems can be broadly classified as follows :

1. Continuous location models (models in plane) are characterised through two essential attributes. (a) The solution space is continuous, that is, it is feasible to locate facilities on every point in the plane. (b) Distance is measured with a suitable metric as the right-angle distance metric or Euclidian metric. The problem then consists in locating one or more facilities in the plane in such a way that

the sum of the weighted distances between customers and their nearest facility is minimised. An alternative objective used within minmax location models is to minimise the maximum (weighted) distance.

2. In network location models distances are computed as shortest paths in a graph. Nodes represent demand points and potential facilities correspond to a subset of the nodes and to points on the arcs. The objective is again to minimise the average or the maximum distance between facilities and customers.
3. Discrete location models or mixed-integer programming models assume that a finite set of potential facility sites is given. The problem consists in finding a subset of facilities and to allocate customer demands to the selected facilities in such a way as to minimise operating costs and allocation costs. Due to their flexibility, mixed-integer programming models are most suitable for the purposes of supporting decision making within distribution system design. Such models are, however, usually complex and very difficult to solve. Sophisticated and highly specialised algorithms are, therefore, required in order to determine optimal or at leat proven near-optimal solutions to practical-sized problems.

For comprehensive surveys on mathematical location-allocation models and their use within distribution planning see, e.g., Aikens (1985), Brandeau and Chiu (1989), Owen and Daskin (2000), Klose (2001), Klose and Drexl (2001b).

Application areas

Applications of facility location models are not restricted to the primary application area of distribution planning. Many other problems where location and allocation decisions are interdependent are covered also. Some of them shall be sketched out as follows :

- Cluster analysis : The topic of cluster analysis is to group items in such a way that items belonging to one group are homogenous and items belonging to different groups are heterogenous. Location then means to select representative items from the overall set of items while allocation corresponds to the assignment of the remaining items to the chosen clusters.

- Location of bank accounts : A company which has to pay suppliers has to decide which bank accounts to use for this purpose in order to optimise float. Cornuejols et al. (1977) consider this problem in detail.
- Vendor selection : Each company must choose vendors for the supply of products. Location in this setting means selecting some vendors from a given set of vendors. Allocation relates to the decision which product to buy from which vendor, see Current and Weber (1994).
- Database location in computer networks : Within a computer network databases can be installed on certain nodes. Installation and maintenance of databases gives raise to fixed cost while transmission times or cost decrease with an increasing number of database installations, see Fisher and Hochbaum (1980).
- Concentrator location : The design of efficient telecommunication and computer networks poses several complex, interdependent problems. In a star-star concentrator network, terminals are connected to a central machine or another (backbone) network via so-called concentrators (see Fig. 1). Determining the layout of such a network is a typical location-allocation problem, see Pirkul (1987) and Chardaire (1999).

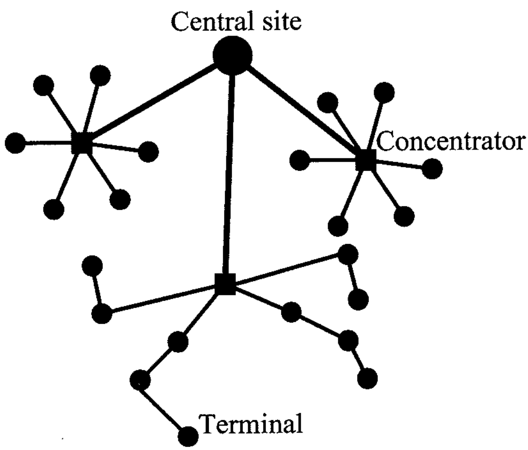

Fig. 1. Star-star concentrator network

- Index selection for database design : Databases comprise a set of tables, each of which consists of several arrays. Relating indices to arrays allows to store entries in a sorted manner yielding fast queries. On the other hand, using indices increases the maintenance time. Caprara et al. (1995) discuss location models for solving this problem.

Algorithms

General-purpose mixed-integer programming (MIP) software can be used to solve discrete location models. In order to solve practical-sized problem instances within a reasonable computation time, however, specialised algorithms which exploit the specific problem structure are required. In the following the main ideas and performance of the developed algorithms are briefly summarised.

An exact algorithm for the Aggregate Capacity Location Problem

The Aggregate Capacity Location Problem (APLP) is to find a subset of facilities and to allocate customer demands to facilities in such a way that total costs are minimised and that the selected facilities have enough capacity to satisfy total demand. The APLP is not important as a stand-alone model but it is an important subproblem of more complex location-allocation models. The developed solution algorithm is based on Lagrangian relaxation, that is selected constraints are relaxed and included in the objective function with a penalty term added. In conjunction with fast methods (dual ascent and subgradient optimisation) for computing suitable "penalty cost coefficients", simple heuristics for determining feasible solutions and extensive problem reduction tests, this allows to find optimal solutions within a branch-and-bound tree in relatively short computation times ; see Klose (1998) and Klose (2001). Figure 2 compares the computation times of this method to those required by a commercial MIP-solver for solving test problems with 1000 customer nodes and 100 potential facility sites.

Column Generation for Capacitated Facility Location

The Capacitated Facility Location Problem (CFLP) extends the APLP by considering capacity constraints for each potential facility. Considering all facility subsets with sufficient capacities and all feasible product

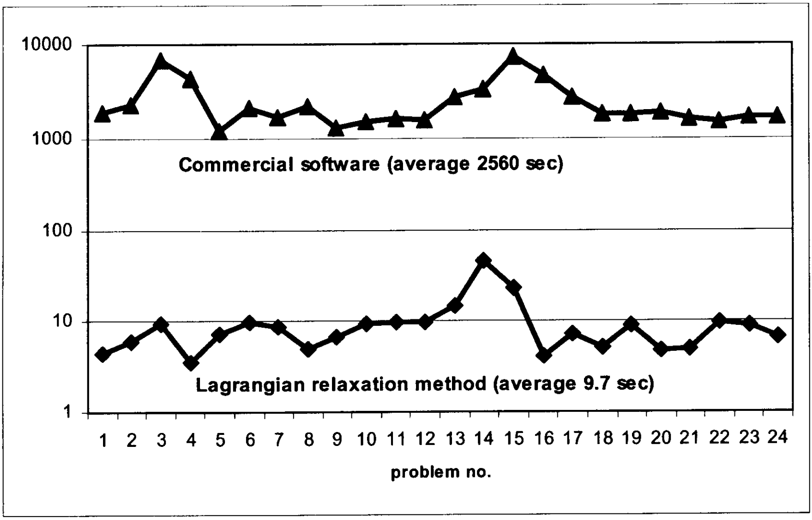

Fig. 2. Comparison of computation time (seconds on logarithmic scale)

flows from depots to customers, the problem can be reformulated as a problem of finding such a subset and product flow such that total costs are minimised and customer demands are met. Column generation techniques are employed for iteratively generating facility subsets and feasible product flows by means of solving sub-optimisation problems. Convergence of this procedure is heavily improved by means of different stabilisation methods. For large problem instances, this allows to compute lower bounds on the optimal solution value within shorter computation times than those required by a linear programming software for solving the linear relaxation of the CFLP. Figure 3 shows a comparison of the lower bounds and the required computation times between the developed column generation algorithm and a linear programming software used to solve the linear relaxation of the CFLP. A detailed description of this method can be found in Klose (2001) and Klose and Drexl (2001a).

A linear programming based heuristic for two-stage capacitated facility location with single-source requirements

In contrast to the CFLP, the two-stage capacitated facility location problem explicitly models the flow of products from a capacity-constrained predecessor stage to the potential depot sites as additional

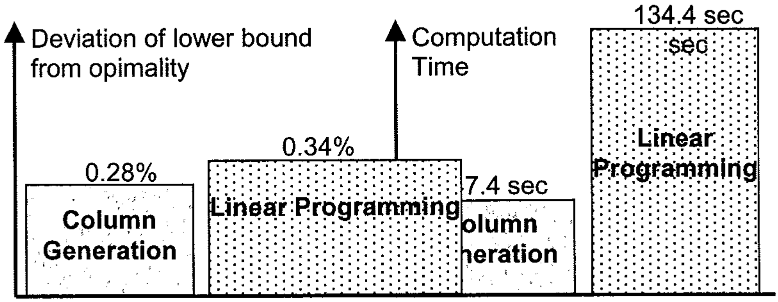

Fig. 3. Comparison between column generation and linear programming for the CFLP

decisions variables. Furthermore, single-sourceing of demand nodes is often required. In case of relatively tight capacity constraints, the linear programming relaxation can usually be solved efficiently. In order to strengthen this relaxation different valid inequalities may be added. Good feasible solutions are obtainable by means of heuristic procedures which search for feasible solutions in the neighbourhood of an optimal solution to the current linear relaxation. Figure 4 illustrates this solution principle. For a number of different test problems with up to 10 plants, 50 potential facility sites and 500 customer nodes this

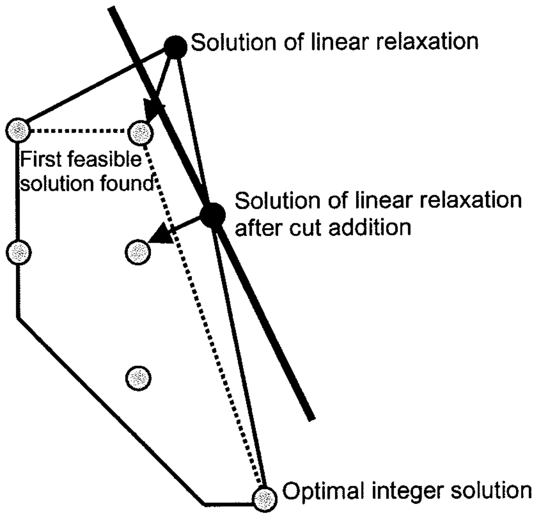

Fig. 4. Linear programming based heuristic

method computed feasible solutions which deviate by 0.5 % from optimality on average (2 % in the worst case). The average computation time was 43 seconds (3 minutes on average for larger test problems and 15 minutes in the worst case). More information about this solution method can be found in Klose (1999) and Klose (2001).

A combined column generation and cutting plane method for two-stage capacitated facility location

Even better feasible solutions and lower bounds for two-stage capacitated facility location problems (with single-source requirements) can be found by combining column generation techniques and cutting plane methods ; see Klose (2000) and Klose (2001). Consider the set of integer solutions satisfying the aggregate capacity requirements, the demand constraints as well as the flow conservation constraints which state that inflow to a depot must equal outflow. The problem then consists in selecting such a solution which minimises total costs and meets the capacity restrictions of the plants and depots. These integer solutions are again iteratively generated by means of solving sub-optimisation problems. Optimal solutions to the linear relaxation of this problem reformulation lie on the intersection of the convex hull of generated

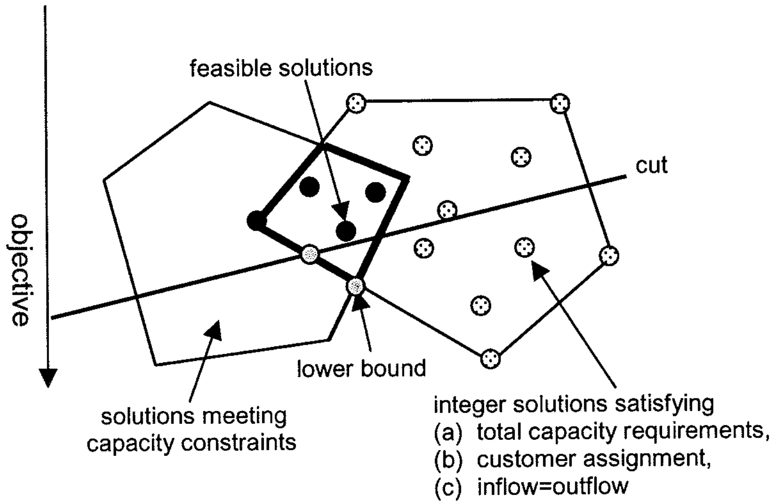

Fig. 5. Combined column generation and cutting plane method

integer solutions and the set of all, possibly fractional solutions meeting the capacity constraints. In order to cut off the resulting, possibly fractional solution valid inequalities can be added. Feasible solutions are found by searching in the neighbourhood of generated integer solutions which violate capacity constraints. Figure 5 illustrates the underlying solution principle. This method generates feasible solutions which deviate from optimality by 0.1 % on average (0.8 % in the worst case). For test problems with up to 10 plants, 50 potential facility sites and 500 customers, the average computation time was 10 minutes (15 minutes in the worst case).

Application to Real-World Problems

The descibed algorithms were employed to support decision making for a number of practical location-allocation problems. These projects were carried out in close cooperation between the firms concerned and the Institute of Operations Research at the University of St. Gallen. In particular, the methods helped

- to determine depot locations for a large Swiss brewery ;
- to locate depots and transshipment points for a Swiss food producer, see Tüshaus and Wittmann (1998) and Klose and Stähly (1998) ;
- to plan locations of training centers for the Federal Office of Civil Protection of the canton Berne ;
- to determine the locations of transshipment points for a wholesaler of newspapers, magazines, food items ;
- to support distribution planning for a Swiss dairy producer, taking into account the option to outsource parts of the secondary distribution, see Engeler et al. (1999) ;
- to find the location of parcel bases, the assignment of parcel bases to parcel centers and the allocation of postcode regions to parcel bases for the Swiss Post, see Bruns et al. (2000).

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