Fault scaling and $1/f$ noise scaling of seismic velocity fluctuations in the Earth's crust

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Ladies and Gentlemen,

This is a very happy moment in my life. Having the opportunity to present some of the results of my research to you on this occasion fills me with joy, satisfaction, and gratitude. Receiving the Latsis award has given me a substantial motivational boost and much needed reassurance and this at an important stage in my career. A stage, when despite an already respectable research, publication, and teaching record, one is often tormented by doubts as to the relevance of ones endeavours. I would therefore like to express my deepest gratitude to the Latsis family and their foundation. It is visionary, community-minded efforts such as the work of the Latsis Foundation that make a decisive difference in a country's scientific culture.

Summary

Sonic logs are detailed measurements of the in situ seismic velocity along borehole walls. Power spectra of sonic logs typically decay approximately as the reciprocal of spatial frequency f, regardless of the chemical composition, geologic age, and tectonic history of the probed lithologies. Data sequences of this type are fractal or scale-invariant. The origins of this uniform $1/f$ scaling of seismic structure are not clear, particularly in low-porosity crystalline rocks, but faults, fractures, and cracks are considered to be important. Fault structures also follow fractal scaling laws and have significant effects on seismic velocity. This paper presents a quantitative model that evaluates the role played by faults in determining the scaling laws of seismic velocity fluctuations. The model is based on current knowledge of the structure and scaling properties of brittle faults and of associated regions of microcracking. By approximating the relationship between crack density and velocity variation as linear, this model yields a Brownian power spectrum ($\propto 1/f^2$) for velocity perturbations across a single fault zone in a medium of otherwise constant velocity. The power spectrum of velocity fluctuations induced by a population of faults is then obtained by superposing the corresponding Brownian power spectra weighted according to the observed frequency-size scaling relationship of brittle faults. The results of this study indicate that the uniform $1/f$ scaling of velocity fluctuations in crystalline rocks seems to be linked to the correspondingly uniform scaling properties of fault structures.

Introduction

Seismic velocities contain information about the chemical composition, physical state, and dynamic evolution of the Earth's interior. In the upper 10 km of the Earth's crust, low frequency seismic observations (~1 to 1000 Hz) and high-frequency laboratory data (~0.1 to 10 MHz) are complemented by sonic log measurements. Sonic logs comprise continuous seismic refraction measurements along the borehole wall with source-receiver spacings of 1 to 3 m and source frequencies of 10 to 50 kHz. Such measurements allow for the determination of in situ velocity-depth profiles along borehole walls with a resolution of about 1 m (Serra, 1984).

Sonic logs are characterized by a high degree of local variability (Figure 1), which can be quantified by basic statistical methods such as the power spectrum. The power spectrum is defined as the square of the amplitude spectrum and thus describes the energy distribution of a data sequence as a function of frequency. All available evidence indicates that over a wide range of spatial frequencies f, sonic log power spectra uniformly decay as $1/f^{\beta}$ (with the exponent β lying between 0.5 and 1.5), regardless of the chemical composition, geologic age, and tectonic history of the probed lithologies (Figure 2; Walden and Hosken, 1985; Todoeschuck and Jensen, 1988; Wu et al., 1994; Bean and McCloskey, 1993; Holliger, 1996). Data sequences of this type are referred to as $1/f$ or flicker noise and are characterized by their fractal nature or scale-invariance (Mandelbrot, 1983). Note that in this context the term "noise" refers to the intrinsic variability of a data sequence and not to any undesired components contained therein.

Scale-invariance is emerging as an underlying property of many phenomena in the earth sciences (Korvin, 1992; Turcotte, 1992; Holliger and Levander, 1994). It is for this reason that every photograph or sketch of a geologic structure must be scaled by an object of defined size, such as a coin or a hammer. Without such scaling, it is generally impossible to determine whether the picture covers less than 1 m or more than 1000 m. Therefore, scaling relations are essential because they allow us to interpolate between observations made at different scales and to extrapolate observations made at a certain scale to other scales (Holliger and Levander, 1994).

The causes of the strong variability of sonic log velocities in low-porosity crystalline rocks remain enigmatic, but several studies suggest that brittle fault structures could be important. Faults, fractures, and cracks have profound effects on seismic velocity (Bourbié et al., 1987) and their scaling relations seem to be as uniform as those of sonic logs (Scholz et al., 1993). Moos and Zoback (1983) found that macrofractures had significant effects on sonic logs, but could not fully explain the observed velocity fluctuations. Unexplained components were attributed to the unknown effects of compositional variations and microfractures. Juhlin (1990) compared porosities measured on granitic core material with corresponding sonic log velocities. He concluded that the low porosities measured could cause the deviations from the average matrix velocity, provided that the pores had aspect ratios of around 0.01 or less. Such low aspect ratios are almost exclusively produced by fractures and cracks. These results were supported and complemented by Leary (1991): for the crystalline rocks penetrated by the Cajon Pass borehole in southern California, he found a strong correlation between sonic and resistivity logs, but no significant correlation between these logs and the corresponding natural gamma logs. Natural gamma logs are primarily sensitive to rock chemistry, whereas resistivity logs are strongly affected by the amount and the salinity of fluids filling pores, cracks, and fractures. Leary (1991) thus argued that seismic velocity fluctuations in low-porosity upper crustal crystalline rocks are dominated by the effects of fracturing. Here, this hypothesis is tested by investigating the compatibility of the $1/f$ scaling of sonic log velocities with the corresponding scaling properties of brittle faults and their surrounding regions of microfracturing.

Velocity Perturbation Across a Brittle Fault Zone

According to Scholz et al. (1993), a brittle fault zone consists of an inner cataclastic region and an outer region where microcracks prevail (Figure 3). Microcracks are induced by stress fields associated with motions of the fault tip, and their density decreases rapidly with increasing distance from the fault plane. Together, the inner cataclastic region and the outer region of microfracturing of a brittle fault are referred to as the damage zone. Although the range of absolute values is not fully constrained, the width of the cataclastic region s is expected to be small in comparison to that of the damage zone P, i.e., s ≈ 10-10000·P (Scholz et al., 1993).

Both observational evidence (Chernyshev and Dearman, 1991; Anders and Wiltschko, 1994) and theoretical considerations (Scholz et al., 1993) suggest that the crack density decays approximately as –log (z), where z is the distance from the fault plane. The negative natural logarithm –log (z) and the zero-order Bessel function of the second kind K_0 (z) converge asymptotically for small values of z and are similar over a wide range of z values. For reasons of mathematical convenience, the following derivations use K_0 (z) rather than –log (z). Given the uncertainty associated with observed microcrack density profiles, this approximation is justified (Anders and Wiltschko, 1994).

To a first approximation, velocity perturbations induced by microcracks depend linearly on the crack-induced porosity, and thus on the crack density (Bourbié et al., 1987). This approximation is equivalent to the empirical linear velocity-porosity relationship commonly used for sedimentary rocks (Wyllie et al., 1956; Telford et al., 1990). A velocity perturbation profile $g(z,P)$ across a brittle fault with a damage zone of width P in a medium of otherwise constant velocity can thus be defined by

$$g(z,P) = -\Delta v \frac{K_0(z/P)}{K_0(s/P)}, \tag{1}$$

where Δv is the velocity perturbation at the edge of the inner cataclastic region ($z = s$). The purpose of the denominator in equation 1 is to scale $g(z,P)$ to the value of at $z = s$. The singularity at $z = 0$ corresponds to the "infinite" crack density (or porosity) in the innermost cataclastic region of the fault zone.

The power spectrum $G(f,P)$ of the velocity perturbation $g(z,P)$ across a brittle fault is given by (e.g., Frankel and Clayton, 1986)

$$G(f,P) = \left(\frac{\Delta v \cdot P}{K_0(s/P)}\right)^2 \cdot \frac{1}{1+(2\pi fP)^2} = \frac{2P\sigma^2}{1+(2\pi fP)^2}, \tag{2}$$

where σ corresponds to the standard deviation of the fault-related velocity perturbation. For $2\pi fP>>1$, this power spectrum decays as $1/f^2$, which is characteristic of so-called Brownian or "brown" noise, whereas for $2\pi fP<<1$, the spectrum is constant or "white." The transition from a brown to a white noise power spectrum occurs around $2\pi fP=1$. The width of the damage zone P thus determines the spatial frequency ranges over which brown or white noise prevails and is also referred to as the correlation scale. Three band-limited Brownian power spectra with correlation scales P of 1, 10, and 100 m are shown in Figure 4.

Velocity Statistics Resulting from Numerous Faults

The above results allow us to derive the power spectrum of velocity perturbations induced by realistic populations of brittle faults, provided that their distribution and scaling statistics known. To this end, the hypothetical situation of a vertical borehole penetrating several horizontal faults is considered. There is evidence that the width of the damage zone P scales approximately linearly with the length L of the fault (Scholz, 1987; Chernyshev and Dearman, 1991; Scholz et al., 1993)

$$L \propto P, \tag{3}$$

whereas the number of faults N within a given volume is approximately inversely proportional to the length L of these faults (Scholz et al., 1993)

$$N(L) \propto {}^{1}\!/_{L}. \tag{4}$$

This implies that the damage zone widths P have a hyperbolic statistical distribution $\rho(P)$ of the form

$$\rho(P) \begin{cases} \propto {}^{1}\!/_{P} & \text{for} \quad P_{\min} \leq P \leq P_{\max} \\ = 0 & \text{otherwise} \end{cases}, \tag{5}$$

where P_{min} and P_{max} are the minimum and maximum damage zone widths present in the fault population considered. The power spectrum of velocity fluctuations due to numerous faults $\bar{G}(f)$ is given by the sum of the power spectra of the individual faults $G(f,P)$ (equation 2) weighted by their statistical distribution $\rho(P)$ (equation 5) (Walden and Hosken, 1985; Schroeder, 1991)

$$\bar{G}(f) = \int_{P_{min}}^{P_{max}} G(f,P)\cdot\rho(P)dP \propto \int_{P_{min}}^{P_{max}} \frac{2P}{1+(2\pi fP)^2}\cdot\frac{1}{P}dP$$

$$= \frac{1}{\pi f}\arctan(2\pi fP)\Big|_{P_{min}}^{P_{max}} = \frac{1}{\pi f}\left[\arctan(2\pi fP_{max}) - \arctan(2\pi fP_{min})\right]. \quad (6)$$

It can be shown that in the frequency interval $1/\pi P_{max} < f < 1/4\pi P_{min}$, the expression $[\arctan(2\pi fP_{max})-\arctan(2\pi fP_{min})]$ is roughly constant, so that $\bar{G}(f) \propto 1/f$. For $f < 1/4\pi P_{min}$, $\bar{G}(f)$ decays approximately as $1/f^2$ (brown noise), and for $f < 1/\pi P_{max}$ it tends toward a constant value (white noise).

The power spectrum $\bar{G}(f)$ of velocity perturbations associated with numerous faults thus approximates band-limited $1/f$ noise with a correlation scale corresponding to about twice the maximum damage zone width ($2P_{max}$). Figure 5 shows the power spectrum $\bar{G}(f)$ resulting from the weighted superposition (equations 5 and 6) of the three band-limited Brownian power spectra with correlation scales P of 1, 10, and 100 m shown in Figure 4. As predicted, this spectrum emulates band-limited $1/f$ noise with a correlation scale of 200 m (twice the maximum correlation scale of the superposed Brownian power spectra). That only three Brownian power spectra need to be superposed to emulate a $1/f$ power spectrum over a wide range of frequencies illustrates the robustness of this model.

So far, it has been assumed that all fault zones lie in planes perpendicular to the borehole axis. To complete this discussion, the effects of variable orientations of the faults need to be considered. Simple geometric considerations demonstrate that the effective width of a dipping fault zone intersected by a vertical borehole is given by $P_{effective} = P/\cos\alpha$, where α is the dip angle of the fault with respect to the horizontal. The model presented here remains valid in its general form, provided that the effective widths of the fault damage zones retain a hyperbolic distribution ($\rho\ (P_{effective}) \propto 1/P_{effective}$). This condition is fulfilled, for example, if all faults have the same orientation or if they are randomly oriented. Both uniform and random distributions of fault orientations are quite common in nature. The case of a uniform orientation of faults is known to be relevant in weakly deformed sedimentary rocks and in crystalline rocks in certain parts of mountain ranges (Price and Cosgrove, 1990). Conversely, in rock formations that have undergone multiple phases of tectonic deformation, several families of faults may be present, such that the net fault distribution appears to be random (Barton and Zoback, 1992).

Discussion

Given both the complexity of upper crustal crystalline rocks and the simplicity of this model, the robustness of the latter is important. One indication of robustness is the model's rapid convergence (Figures 4 and 5). Another indication is how deviations from the perfectly hyperbolic distribution of the effective damage zone widths (equation 5) affect the resulting power spectrum of the velocity fluctuations. To address this question, power spectra of the type given by equation 6 have been calculated using hyperbolic distributions of correlation scales P contaminated by both systematic and random biases. The results indicate that the $1/f$ decay of the resulting power spectra is a remarkably stable feature. This is compatible with findings that data sequences with imperfect hyperbolic distributions, such as lognormal distributions, may still exhibit $1/f$ noise behavior over a wide range of frequencies (West and Shlesinger, 1990). This robustness, combined with nature's tendency towards quasi-hyperbolic distributions are likely causes of the widespread occurrence of 1/f scaling phenomena.

The primary purpose of this paper is to investigate the statistical uniformity of sonic log fluctuation in low-porosity crystalline rocks. However, power spectra of sonic logs from sedimentary rocks show a comparably uniform $1/f$ decay (Walden and Hosken, 1985). Given that macrofractures and microfractures are ubiquitous, even in essentially undeformed sedimentary rocks (Price and Cosgrove, 1990), one can expect that the arguments made in this paper are at least partially applicable to seismic velocity fluctuations in sediments. For example, Bean and McCloskey (1993) attribute the $1/f$ scaling of sonic log data measured in chalk layers offshore Ireland primarily to the fracturing of these lithological units.

Although tip stresses of brittle faults are undoubtedly important, there are other mechanisms that generate microcracks, such as distributed brittle strain or differential thermal expansion of individual mineral components. At present, little is known about the statistical distribution of microcracks that are not related to fault zones. It is, however, reasonable to assume that corresponding crack densities Φ follow some form of a power law distribution $\rho\ (\Phi) \propto 1/\Phi^{\alpha}$ (i.e., small crack densities are much more common than large ones) and hence are characterized by a power spectrum of the form $G\ (f) \propto 1/\Phi^{\alpha}$ (Bak et al., 1987). To comply with the $1/f$ scaling of sonic logs, crack densities would have to follow an approximately hyperbolic (or lognormal) distribution (i.e., $0.5 < \alpha < 1.5$). Laboratory experiments suggest that microcracks caused by distributed brittle strain tend to exhibit distributions of this type (Hirata et al., 1987). Furthermore, there are theoretical arguments that imply that fracture processes spontaneously evolve into barely stable, critical states (self-organized criticality). This would also imply a quasi-hyperbolic distribution of crack densities (Bak et al., 1987; Bak and Tang, 1989; Sornette et al., 1990). Crampin (1994) explained the uniform seismic shear-wave anisotropy in the upper crust by assuming that all rocks tend to be in a similar state of fracturing (fracture criticality). According to this model, the range of crack densities between the extreme, and rarely observed, states of totally intact and totally disintegrated brittle rocks is similarly narrow for most rock types. Crampin's (1994) concept of fracture criticality thus implies a uniform, albeit unspecified, statistical distribution of microfractures and macrofractures throughout the uppermost crust. Qualitatively, this is compatible with the model presented in this paper as well as with the narrow range of correlation scales and standard deviations characteristic of upper crustal velocity fluctuations (Holliger, 1996).

Despite the poor correlation of upper crustal sonic log velocities with rock chemistry (Leary, 1991; Holliger, 1996), the influence of compositional variations on seismic velocity structure cannot be ignored completely. Upper crustal gneisses are often characterized by compositional banding over a wide range of scales. If the individual compositional bands have significantly different seismic velocities (e.g. due to interlayering of mafic and felsic components) and if the distribution of layer thicknesses is approximately hyperbolic, the induced velocity perturbations are expected to exhibit 1/f scaling. The same arguments apply for velocity fluctuations in finely layered sedimentary rocks (Walden and Hosken, 1985).

Conclusions

This paper has investigated the relation between the scaling laws of upper crustal seismic velocity fluctuations evidenced by sonic logs and the scaling laws of brittle fault structures. To this end, a quantitative model of the power spectra of velocity perturbations induced by brittle faults and their damage zones has been developed. The results of this study indicate that the uniform $1/f$ scaling of seismic velocity variations in low-porosity crystalline rocks is compatible with the similarly uniform scaling of faults and their surrounding regions of microfracturing. This illustrates that the scaling laws of secondary phenomena (e.g., seismic structure) may contain critical information about the fundamental processes that have generated them (e.g., brittle deformation).

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Figure Captions

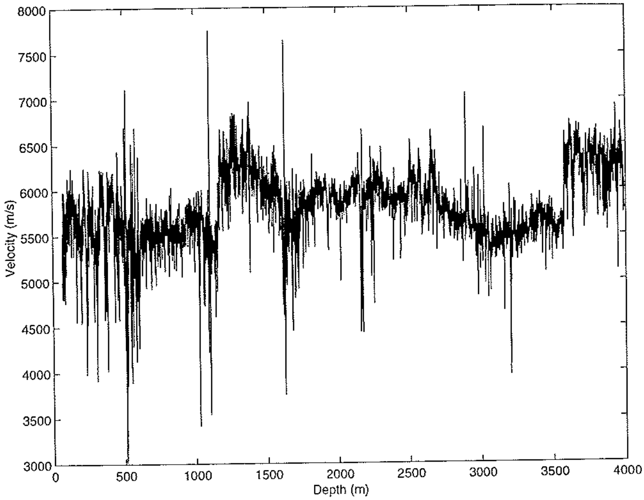
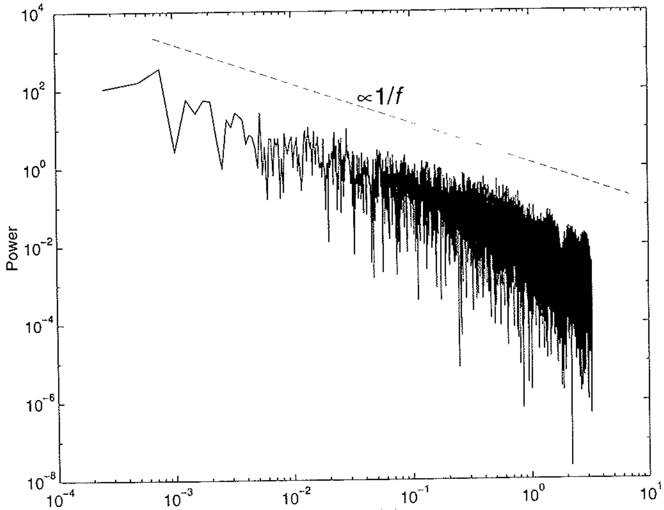


Figure 1. P-wave sonic log from the pilot borehole of the German Deep Drilling Program (KTB).

Figure 2. Power spectrum of the P-wave sonic log from the KTB pilot borehole. The dashed line shows the slope of an ideal $1/f$ noise power spectrum.



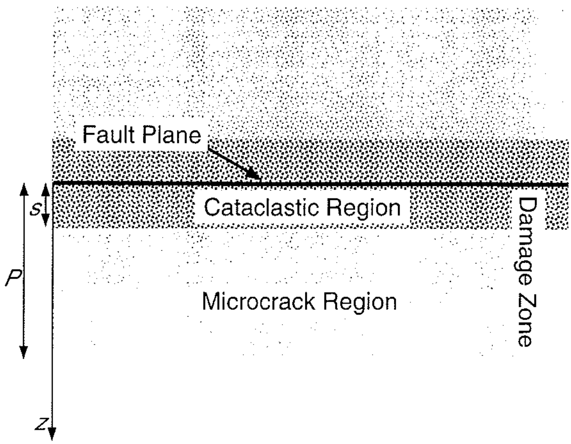
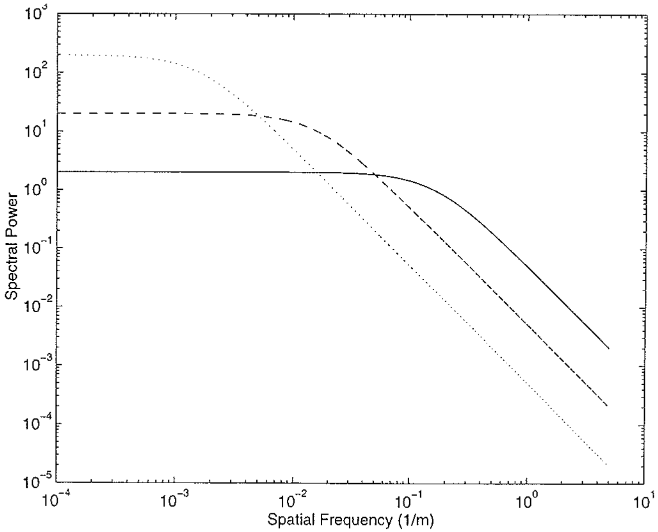


Figure 3.
Schematic model of a typical brittle fault zone consisting of the inner cataclastic region and the outer microcrack region (Scholz et al., 1993). The sketch is not drawn to scale, because the thickness of the damage zone P is expected to be much larger than that of the cataclastic zone s (s ≈ 10-10000·P). Note that the damage zone comprises both the inner cataclastic region and the outer microcrack region.

Figure 4.
Band-limited Brownian power spectra (see equation 2) with correlation scales of 1 (solid line), 10 (dashed line), and 100 m (dotted line).



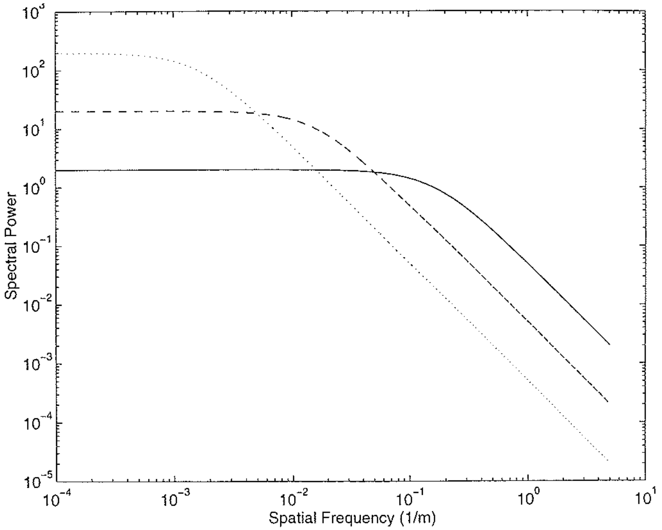


Figure 5.
Solid line: power spectrum resulting from the weighted superposition of the three band-limited Brownian power spectra shown in Figure 4 (see equations 5 and 6); dashed line: power spectrum of band-limited $1/f$ noise, i.e., $\propto 1/[1+(2\pi fP)^2]^{1/2}$, with a correlation scale P of 200 m. Both power spectra are normalized for display purposes.